Sr. No. 2598

Exam Code: 103202

Subject Code: 7437

B.A./B.Sc. - 2nd Sem. (Old Syllabus 2015)

(2519)

Paper: Mathematics Paper-I

(Integral Calculus & Differential Equations)

Time allowed: 3 hrs. Max. Marks: 50

Note: Attempt five questions in all, selecting at least two questions from each section.

Section-A

1. a) Evaluate $\int \frac{\text{Sinhx}}{\text{Sinh}^3\text{x} - \text{Cosh}^3\text{x}}\,\text{dx}$

   b) If $I_n = \int_0^{\pi/2} x^n \sin x\,dx$ and n>1 prove that $I_n + n(n-1)\,I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$

   Hence evaluate $\int_0^{\pi/2} x^4 \sin x\,dx$ (5+5=10)

2. a) Prove that $1 \le \int_0^{\pi/2} \sqrt{\sin x}\,dx \le \frac{1}{2}\sqrt{2\pi}$

   b) Evaluate $\frac{(n!)^{\frac{1}{n}}}{n}$ when $n \to \infty$ (5+5=10)

3. a) Show that $\int_0^{\pi/2} \frac{\cos^2 x}{\sin x + \cos x}\,dx = \frac{1}{\sqrt{2}} \log(\sqrt{2}+1)$

   b) Find the volume of the Solid formed by the revolution about x-axis of the loop of the curve $Y^2 = \frac{x^2(3a-x)}{a+x}$ where $x \neq -a$

   (5+5=10)

4. a) Find the area of the region bounded by the curve $y=x^2+1$, $y=x$, $x=0$ and $y=2$.

   b) Prove that

   $$\frac{2}{\pi}\int_0^{\pi/2} \frac{dx}{(1-e^2\sin^2 x)^{\frac{1}{2}}} = 1 + \frac{1^2}{2^2}e^2 + \frac{1^2.3^2}{2^2.4^2}e^4 + \frac{1^2.3^2.5^2}{2^2.4^2.6^2}e^6 + \ldots\ldots\ldots$$

   where e>1 (5+5=10)

5. a) State and prove fundamental Theorm on Integral Calculus.

   b) Find the length of a loop of the curve

   $9ay^2 = x(x-3a)^2$, a>0 (5+5=10)

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Section-B

6. a) Solve $(3x^2y^4+2xy)dx + (2x^3y^3-x^2)dy=0$

   b) Solve $y-2px = \tan^{-1}(xp^2)$

   where $p=\frac{dy}{dx}$ (5+5=10)

7. a) Find the singular solution of the equation $8p^3x=y(12p^2-9)$

   b) Using method of variation of parameters

   Solve $(D^2-1)y = 2(1-e^{-2x})^{\frac{-1}{2}}$ where $D = \frac{d}{dx}$

8. a) Solve the differential equation

   $$(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2sin(log(1+x))$$

   b) Solve $\frac{d^2y}{dx^2} + 16y = sec4x$ (5+5=10)

9. a) Solve the differential equation

   $$\frac{d^2y}{dx^2} + y = x\,e^x sin2x$$

   b) Solve the differential equation

   $$\sqrt{x}\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 3y = x, x > 0$$ (5+5=10)

10. a) Solve in series

    $$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 20y = 0$$

    b) Prove that

    $\frac{1}{D-\alpha}Q = e^{\alpha x}\int Qe^{-\alpha x}dx$, no orbitrary constant being added.

(7+3=10)

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